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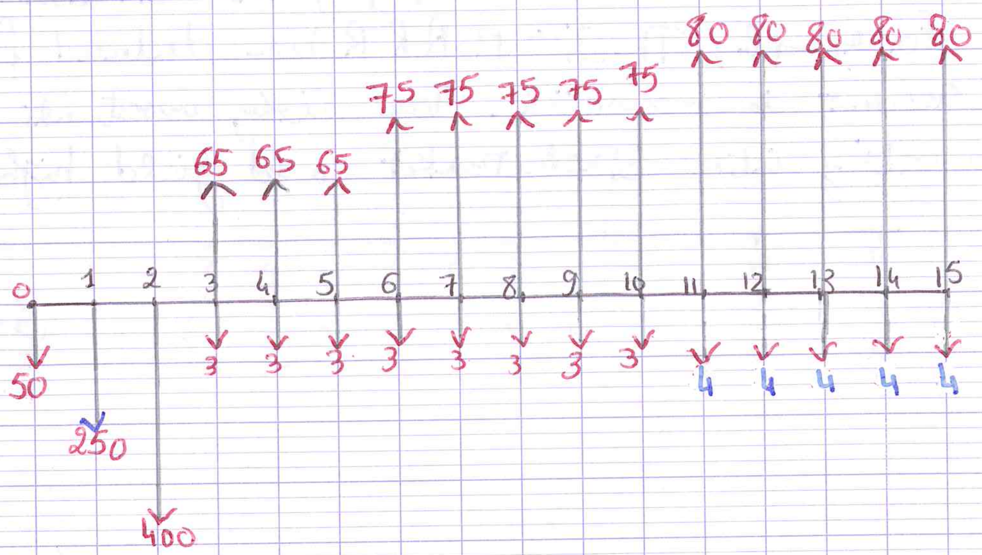
INDE 301: Homework

Prob I: (Highway): Annual discount rate 10% $\Rightarrow$ MARR = 10% per year



AW = AW of Benefits − AW of Costs

= AWB − AWC

Study Period = 15 years

$$AWB = \{P\} \times (A|P, 10\%, 15)$$

$$AWB = \{65(P|A, 10\%, 3)(P|F, 10\%, 2) + 75(P|A, 10\%, 5)(P|F, 10\%, 5) + 80(P|A, 10\%, 5)(P|F, 10\%, 10)\} \times (A|P, 10\%, 15)$$

AWB = 56.1 millions of dollars.

$$AWC = \{P\}(A|P, 10\%, 15)$$

$$= \{50 + 250(P|F, 10\%, 1) + 400(P|F, 10\%, 2) + 3(P|A, 10\%, 8)(P|F, 10\%, 2) + 4(P|A, 10\%, 5)(P|F, 10\%, 10)\} \times (A|P, 10\%, 15)$$

AWC = 82.4 millions of dollars

$AW = AWB - AWC = 56.1 - 82.4 = -26.3$ Millions

$AW < 0 \Rightarrow$ the investment is not attractive.

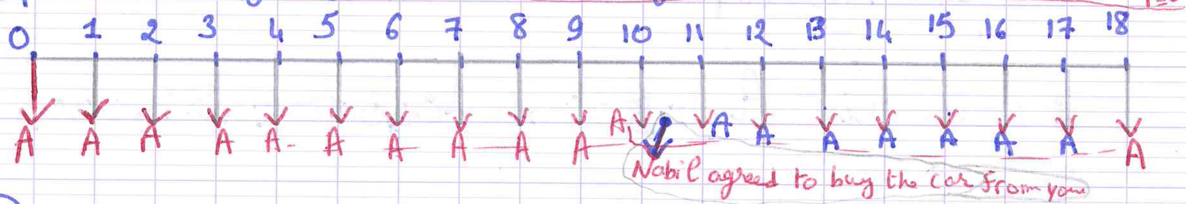
Problem 2:

Interest rate : $\underbrace{12\%}_{\text{Nominal: } r}$ per year Compounded $\underbrace{\text{monthly}}_{\text{Compounded period}}$ $\Rightarrow m = 12$ months / year

Effective interest rate per Compounded period $= \dfrac{r}{m}$

Effective interest rate per month $= \dfrac{12\%}{12} = 1\%$.

19 equal monthly Payments with the first payment as down payment $\Rightarrow$ at $t=0$



Price $= 12000$ \$

$$\text{Price} = A + A \times (P/A, \underbrace{1\%}_{\text{Effective interest rate / month}}, \underbrace{18}_{\text{number of months}})$$

$$12000 = A\left(1 + (P/A, 1\%, 18)\right) \Rightarrow A = \frac{12000}{(1 + 16.3983)}$$

$$\boxed{A = \$689.70}$$

b) After 11 payments (the down payment and ten other payments)

In this question the present is at $n = 11$

Nabil will pay the remainder of the loan starting at $n = 11$

$$\text{Payoff} = A \times (P/A, 1\%, 7) + 689.70 \$$$

He will pay month $11 \rightarrow 18$

$$= 689.70 + 689.70 \times 6.7282$$

$$\boxed{\text{Payoff} = \$5330.1}$$

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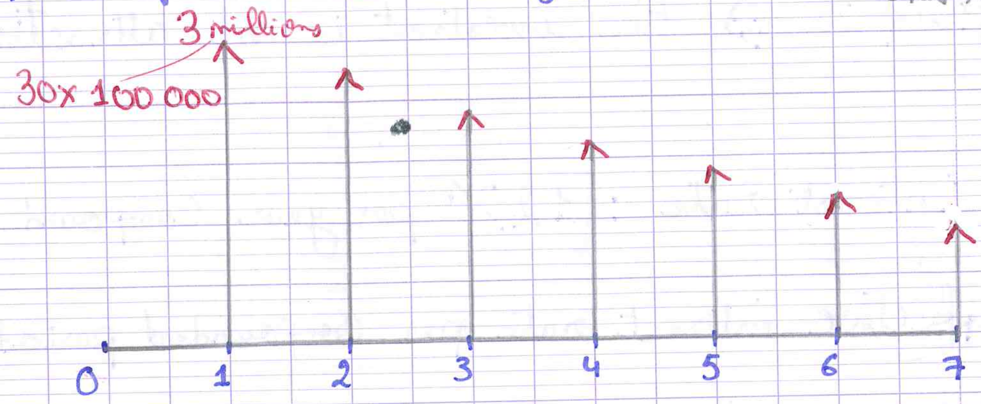
Problem 3: Interest rate is 12 % per year

Price of oil : 30 $ during the first year

In This statement the is no compounding Period, we consider this rate as effective per year



Production is expected to decrease by 10 %

a) The expected revenue for the first year of production

is $A_1 = \$30 \times 100000 = \$3{,}000{,}000$

The revenue stream follows a decreasing geometric gradient series having a rate $\boxed{g = -10\%}$. {Geometric gradient series}

$i = 12\%$ $\quad g = -10\%$ $\quad i \neq g$, $n = 7$

PW of the well $= P_g = A_1 \times (P|A_1, -10\%; 12\%, 7)$

$$P_g = A_1 \times \left[\frac{\left(1 - \left(\frac{1+g}{1+i}\right)^n\right)}{(i - g)} \right]$$

$$= A_1 \times \left[\frac{1 - \left(\frac{1 - 10\%}{1 + 12\%}\right)^7}{12\% - (-10\%)} \right] = 3000000 \times 3.562$$

$P_g = 10{,}686{,}037.81$

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b) Price of oil starts at \$30 (first year) but to increase 5% per year for the next seven years.

In this case, the oil price increases at the annual rate of 5% while the oil production decreases at annual rate of 10%.

Therefore the annual revenue in year n can be expressed

as $$A_n = A_{n-1} \underbrace{(1+5\%)}_{\text{Price increasing}} \underbrace{(1-10\%)}_{\text{Production decreasing}}$$

$$A_n = A_{n-1} \times 0.945$$

This revenue series is equivalent to a decreasing

Geometric series with $\boxed{g = -5.5\%}$

The present worth of this new series is:

$$P_g = A_1 (P|A_1, g, i, n)$$

$g = -5.5\%$
$i = 12\%$
$n = 7$
$i \neq g$

$$= 3000000 \times \left[\frac{1 - \left(\frac{1+g}{1+i}\right)^n}{i-g}\right]$$

$$= 3000000 \times \left[\frac{1 - \left(\frac{1-5.5\%}{1+12\%}\right)^7}{12\% - (-5.5\%)}\right]$$

$$= 3000000 \times 3.9746$$

$$\boxed{P_g = \$11,923,948.35}$$

c) A fair price would equal the present worth of the remaining series (A_4, A_5, A_6, A_7) at the end of Year 3

$$P_g = A_4 \left(P|A_4, g, i, n\right)$$

$$\boxed{P_g = A_4 \times \left(P|A_4, -5.5\%, 12\%, 4\right)}$$

$g = -5.5\%$
$i = 12\%$
$n = 4$ $\quad i \neq g$

$$A_n = (1+g)^{n-1} \times A_1$$

$$A_4 = A_1 \times (1+g)^3$$

$$A_4 = A_1 (1-5\%)^3 = 3000\,000 \times (0.945)^3$$

$$A_4 = 2{,}531{,}725.88$$

$$P = 2{,}531{,}725.88 \times \left[\frac{1 - \left(\frac{1-5.5\%}{1+12\%}\right)^4}{12\% - (-5.5\%)}\right]$$

$$= 2\,531\,725.88 \times (2.8182)$$

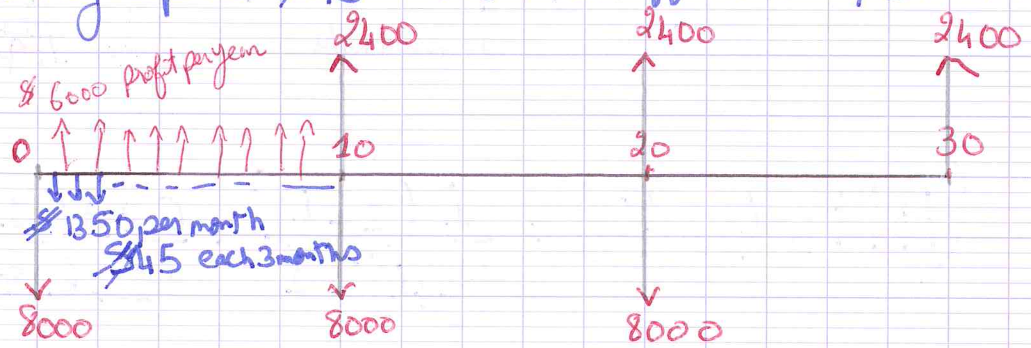
$$\boxed{P = \$7{,}134{,}813.91}$$

The Fair Price of the well

Problem 4: Taxi Driver

Monthly expenses: $1200 + 150 = \$1350$ with i = effective i/month

Quarterly expenses: \$45 with i = effective i/3 months



Gorges wants to make annual revenue (profit) of \$ 6000/year

Interest rate: 6 % year compounded monthly:

Find effective i per month, per 3 month, per year:

Effective/month: $i_1 = i_{\text{Compounded period}} = \frac{6\%}{12} = 0.5\%$ per month

Effective/3months: $i_2 = \frac{i}{1} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.015}{3}\right)^3 - 1$:

$i_2 = 1.51\%$ per 3 months.

Effective Annual interest rate $= \left(1 + \frac{\overset{\text{nominal per year}}{r}}{m}\right)^m - 1$

$$i_3 = \left(1 + \frac{6\%}{12}\right)^{12} - 1 = 6.17\% \text{ per year}$$

In this example, we can use the Annual worth analysis for one life-cycle and it will be the same for three life-cycle

Effective annual interest rate

10 years = One life cycle

$$\text{Expenses: } AW = -8000 \times (A|P, i_3, 10) + 2400 (A|F, i_3, 10)$$
$$- 1350 (F|A, \underbrace{i_1, 12}_{12 \text{ months/year}}) - 45 (F|A, \underbrace{i_2, 4}_{4 \text{ quarters/year}})$$

$$(A|P, i_3, 10) = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{6.17\%(1+6.17\%)^{10}}{(1+6.17\%)^{10} - 1}$$
$$= 0.136964$$

$$(A|F, i_3, 10) = \frac{i}{(1+i)^n - 1} = \frac{6.17\%}{(1+6.17\%)^{10} - 1}$$
$$= 0.07526$$

$$(F|A, i_1, 12) = \frac{(1+i)^n - 1}{i} = \frac{(1+0.5\%)^{12} - 1}{0.5\%}$$
$$= 12.3356$$

$$(F|A, i_2, 4) = \frac{(1+i)^n - 1}{i} = \frac{(1+1.51\%)^4 - 1}{1.5\%}$$
$$= 4.0915$$

Expenses: $AW = \$-17752.26$ (Total costs)

To ensure \$6000 (of profit per year, Therefore income should be at least $17752.26 + 6000$

$\boxed{\text{Income Should be at least} = 23752.265 \text{ /year}}$

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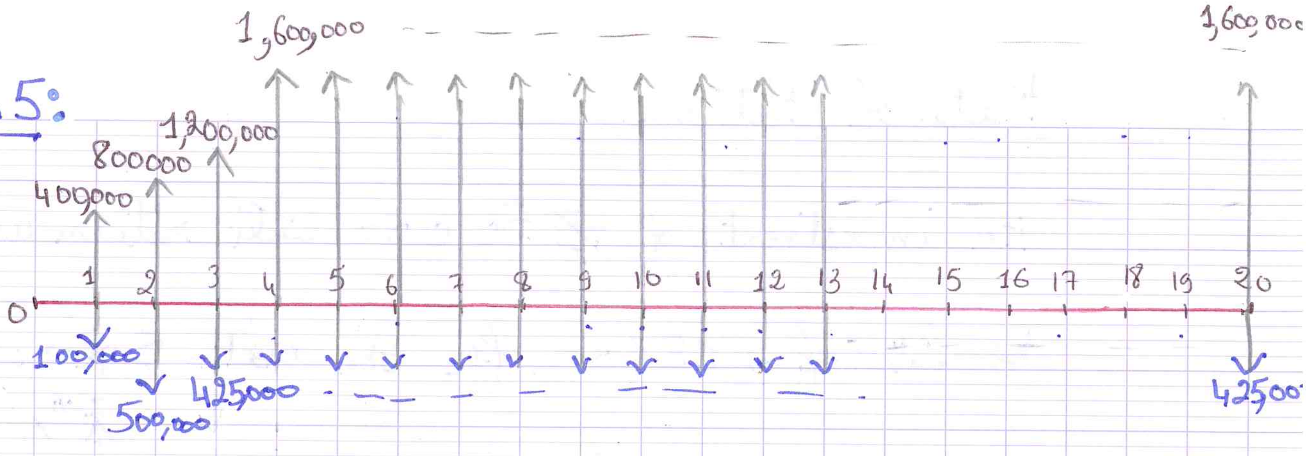
Problem 5:



First year expenses: $100, 000

year	Cost	Revenue
1	$ 100,000	$ 400,000
2	$ 500,000	$ 800,000
3	$ 425,000	$ 1,200,000
4	$ 425,000	$ 1,600,000
⋮	⋮	
20	$ 425,000	$ 1,600,000

MARR is 10% per year:

Find. AW or PW at MARR, if PW ≥ 0 or AW ≥ 0 the the project is attractive.

$\underline{PW} = 400000(P/F, 10\%, 1) + 800000(P/F, 10\%, 2)$

$+ 1,200,000 \times (P/F, 10\%, 3) + 1,600,000 (P/A, 10\%, 17) \times (P/F, 10\%, 3)$

$- 100,000 \times (P/F, 10\%, 1) - 500,000 \times (P/F, 10\%, 2)$

$- 425,000 (P/A, 10\%, 18) \times (P/F, 10\%, 2)$

at MARR = 10% → $\boxed{PW = \$ 8,184,319.222} > 0$ ⇒ AUB Should introduce IE.